Poisson Excess and Integration

Cascaded model assumes every quanta shows Poisson distribution. However, many physical interactions, including photo-absorption, are not based on Poisson process. Thus, I. A. Cunningham et. al. introduced a workaround term: Poisson excess [1].

$$\epsilon_m = \frac{\sigma_m^2}{\bar{m}} - 1$$

The Poisson excess term applies to any random variable in the Cascaded System not only quanta itself but also amplification ratio of a stage, i.e. photo-intensity attenuation in a photo-detector material which often modeled as a binomial selection. If a photo-detector has a ratio of input-to-output photo-intensity, or transmittance (T_p) the photo-detector can be modeled as a gain stage with gain of T_p and the gain variance of $T_p(1-T_p)$ resulting the system equation of:

$$q_{n+1} = T_p q_n$$

$$NPS_{n+1} = T_p^2 NPS_n + q_n T_p (1 - T_p)$$

In this case, the Poisson excess term ($\epsilon_{T_p} = -T_p$) and the noise power spectra can be re-written as,

$$NPS_{n+1} = T_p^2 NPS_n - q_n \epsilon_{T_p} (1 + \epsilon_{T_p})$$

In other words, we would rather define the Amplification module when the with Poisson Excess term:

$$NPS_{n+1} = \bar{g}_n^{\,2} NPS_n + \bar{q}_n \bar{g}_n (\epsilon_{\bar{g}} + 1)$$

instead of,

$$NPS_{n+1} = \bar{g}_n^{\,2} NPS_n + \bar{q}_n \sigma_g^2 \quad .$$

Physical Variable (i.e. quanta, NPS, etc.)	Poisson Excess (ϵ_m)
Poisson Distribution	0
Fixed Variable	-1
Binomial Selection	$-\bar{g}$

Table 1: A few known examples of Poisson excess.

Table 1 shows a few examples of the Poisson excess term. In short, it is an algebraic trick to adjust noise spectra of a gain stage which simplifies the modularization of a detector model.

The Swank factor is not an exception. The Swank factor depicts reduced absorption efficiency due to noise when the incident photon has extreme energy such as X-rays and can be depicted as [2]:

$$I_m = \frac{m^2}{m^2 + \sigma_m^2}$$

where, m is the absorption coefficient of the photo-detector. The variance of photo-absorption (σ_m^2) is clearly not a Poisson distribution. Therefore, we can define the Poisson Excess term for the Swank factor as,

$$\epsilon_m = \frac{m}{I_m} - (m+1)$$

Refreneces

1. I.A. Cunningham, M. S. Westmore, and A. Fenster, "A spatial-frequency dependent quantum accounting diagram and detective quantum efficiency model of signal and noise propagation in cascaded imaging systems," Med. Phys. 21 (3), pp. 417 - 427 March 1994
2. R. K. Swank, "Absorption and noise in x-ray phosphors," J. Appl. Phys., vol. 44, no. 9, pp. 4199-4203, 1973